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RADICAL REPRESENTATIONS OF SHIP SIMULATIONS.(U)

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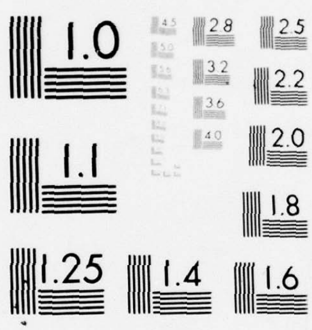

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RADICAL REPRESENTATIONS OF SHIP SIMULATIONS

A. V. HERSHEY

JULY 1977

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July 1977

RADICAL REPRESENTATIONS
OF
SHIP SIMULATIONS

By

A. V. HERSHEY

Science and Mathematics Research Group

(See 1473)

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FOREWORD

In this report there is offered an improved method for the fairing of ship lines, for which a need has been indicated by experience with a computation system for surface wave trains. Illustrative ship plans for tankers have been obtained from Mr. Riesenberger of the Maritime Administration and for warships from Mr. Keane of the Naval Ship Engineering Center. The manuscript for the report was completed by 15 July 1977.

Released by:

Ralph A. Niemann

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ABSTRACT

Straight lines, circles, and splines are used in most mathematical representations of ship lines. The representations are only piecewise analytic. Discontinuities in curvature occur along waterlines and section lines. The discontinuities upset computations of velocity distribution in the flow around the ship. The discontinuities are smoothed when the lines are approximated by orthonormal polynomials. The discontinuities are eliminated when the lines are simulated with the aid of radicals. A smooth simulation of the Series 60, Block 0.60 ship model is given by a radical representation.

INTRODUCTION

The traditional representation of a body of revolution is a table of radial distances from the axis of symmetry at a series of stations along the axis of symmetry. The traditional representation of a foil or a ship is a *table of offsets* at a rectangular grid on the median plane of the foil or the ship. The offsets for the ship define the waterlines and the section lines on the surface of the ship.

An introduction to the mathematical representation of the surface of a ship has been published in a book by Kuo[1]. Included in the book are analyses of the curved sections of the ship from stem to stern. Reviews are given for some published methods for the fairing of ship lines. Further analyses of ship lines have appeared in more recent publications[2-9].

The flow over a body has been simulated by Hess and Smith[12-14] with a source distribution over a polyhedron whose vertices lie on the surface of the body. The normal and the area at the surface are assumed to be the normal and the area at a face of the polyhedron. The source density is assumed to be uniform over each face of the polyhedron, and the velocity in an ideal flow over the polyhedron is taken to be the velocity at the midpoint of each face. Accurate agreement between the polyhedral simulation and an analytical representation is possible for a polyhedron with a large enough number of faces. The velocity for potential flow over a real polyhedron would be infinite at each edge of the polyhedron. The flow over a dihedral angle is analysed in Appendix C.

For an irregular shape it would be necessary to use a polyhedral simulation, but for a smooth shape it should be possible to take advantage of the smoothness in order to increase accuracy and reduce cost. The interaction between two points in an unbounded fluid is given by the simple inverse square law, whereas the interaction under a free surface is given by the derivatives of an expensive Havelock integral. A reduction of cost by the use of fewer data is vital for surface ships.

It has been the goal of naval architects to achieve smooth waterlines. Smoothness was achieved by eye. The lines of a ship were laid out in a mold loft, where ducks* and splines were used in fairing the lines. A duck is a lead weight with a protruding tongue, and a spline is a slender strip with a groove along the edge. The tongue of the duck engages the groove of the spline and holds it in place. The tongue of the duck is finite in breadth. Translation and rotation of the duck controls the position and the slope of the spline. The curvature is not continuous across the node where the duck is in contact with the spline.

Much attention has been devoted to mathematical splines in the literature. An analysis is to be found in Hildebrand's *Introduction to Numerical Analysis*[11]. The length of a curve is subdivided into intervals. The ordinate in each interval is expressed in terms of the abscissa by a cubic polynomial. Inasmuch as the cubic polynomial has four coefficients, it can be made to meet four conditions. It can be made to have specified ordinates at the two ends of the interval. It can be made to maintain continuity of first and second derivatives across boundaries between intervals. The coefficients of the spline approximations are the solution of a tridiagonal system of equations. Recurrence expresses each successive approximation in terms of a pair of parameters which are determined by conditions at the ends of the curve.

*Spline weights are called *dolphins* in Webster's unabridged dictionary!

Linearized cubic splines have been analysed by Mehlum[8]. From the calculus of variations he has shown that the minimization of the square of the second derivative leads to the cubic spline. The elastic curve is the equilibrium configuration of a physical spline. The minimization of the square of the curvature along an elastic curve leads to the special case of a physical spline to which forces are applied but not torques. An analysis of the physical spline is given in Appendix D.

Minimization of the square of the curvature has been used for smoothing in the AUTOKON system by Mehlum. For the representation of smooth lines he has used sequences of circular arcs. Such representations have application in the fabrication of ships where flame cutters are guided along straight lines or along circles. Let the successive points along a curve be joined by chords which span the intervals between the points. Continuity of ordinates from interval to interval is achieved if the chords are spanned by circles with centers on the perpendicular bisectors of the chords.

The circular arc between two points and the radii to the ends of the arc are the boundaries of a sector of a circle. Continuity of slopes from interval to interval is achieved if the centers of the circles are located at the intersections between boundary radii and perpendicular bisectors. The representation of a curve as a sequence of circular arcs can be constructed in a straightforward progression in terms of the radius of the first circle. The construction of a smooth curve requires an adjustment of the first radius by iteration. That the AUTOKON system has been successful for the fairing of 150 ships has been reported by Mehlum. That special skill by an operator is required in the fairing process has been reported by Doornbos[9].

Any simulation with the aid of splines can be only piecewise analytic. Furthermore, freedom from the limitations of traditional methods is possible now that plotters and machine tools can be driven by servo motors under the control of computers.

A conformal mapping of a section line on a circle has been proposed by von Kerczek and Tuck[6]. Their representation of a section line is just an expression of Cartesian coordinates in terms of a parametric angle by trigonometric polynomials. Inasmuch as the representation is horizontal at the base line and is vertical at the waterline, it is not suitable for most section lines.

A mathematical definition of a surface is obtained when Cartesian coordinates of the surface are expressed as parametric functions of surface coordinates. The representation of Cartesian coordinates in terms of surface coordinates makes possible the computation of normals and areas. A quadrilateral boundary on the curved surface can be mapped into a square boundary on a plane map. The connection between Cartesian coordinates on the quadrilateral and the surface coordinates in the map is indeterminate. A point on the surface remains on the surface for any arbitrary transformation which moves the point parallel to the surface.

A study of orthogonal transformations has been made by Miloh and Patel[7]. They proposed trajectory integration to trace lines which are orthogonal to section lines. They considered also an identification of coordinate lines with traces of minimum and maximum curvature. Such an identification would give a unique mapping transformation, but they found that the coordinate lines would have awkward curvatures.

The connection between Cartesian coordinates and map coordinates is determinate when the map transformation is isometric. The Cartesian coordinates are expressed in terms of distance along lines in a network on the surface. An isometric mapping was used already in 1960 for the Taylor Standard Ship[17]. The use of distance along a curve as a parametric variable has been proposed again recently by Jancaitis and Junkins[16].

The parametric representations can be expressed with the aid of polynomial approximations. Past experience with higher order polynomials has shown that they are difficult to control when they are based upon data which have equally spaced arguments. They are not difficult to control when they are based upon data which have arguments with the spacing of the roots of a Chebyshev polynomial. Even when the arguments are equally spaced, the polynomials are easy to control when least squares are applied to a large number of closely spaced data.

A mathematical representation of the Series 60, Block 0.60 ship model was reported at the First International Conference on Numerical Ship Hydrodynamics[18]. Data on the hull configuration were derived from a table of offsets in Todd's report[15]. The hull has flat sides and a flat bottom which are joined by a curved surface. The flat bottom intersects the curved surface along a line of tangency. The actual models had various stern configurations with propellers and rudders. Only the simplest configurations at the stem and at the stern are within the scope of the present project. The actual models had a length of 20 ft from fore perpendicular to aft perpendicular. The mathematical model has been scaled to have the same dimensions.

The tabular data in Todd's report are too sparse to define the configuration mathematically. An attempt to force the section lines to be tangent to the flat bottom on the tabulated line of tangency caused irreducible wiggles in a few of the section lines. It was necessary to assume that each section line intersects the bottom at a finite angle. The table of offsets was augmented with additional section lines and additional waterlines near the edges of the sides of the model.

Each side of the hull was simulated by a single quadrilateral which extends from the fore perpendicular to the aft perpendicular. The arguments of polynomial approximations were proportional to the longitudinal distance to a cross section, and to the lateral distance along the section line. The table of offsets was adjusted by trial until the waterlines and the section lines were fair in CalComp plots.

Actual waterlines have discontinuities in curvature where the forebody joins the midbody and where the midbody joins the aftbody. Actual section lines have discontinuities in curvature at the fore perpendicular where the straight stem bends outward, at the midship section where the curved bilge joins the straight side, and at the aft perpendicular where the straight stern bulges outward. Section lines near the stern are flat at the bottom, convex at the bilge, concave at the bend, and convex at the bulge. Associated with discontinuities in curvature are discontinuities in velocity which would disrupt a numerical analysis of velocity. Furthermore, the mathematical model had a dihedral angle where the sides met the bottom. Any potential flow over a convex dihedral angle would have infinite velocity at the vertex of the dihedral angle. This might explain a bizarre velocity distribution which was obtained near the junction between the sides and the bottom.

A more realistic flow field would have separation with the origination of vortex sheets from the vertex of each dihedral angle. The analysis of flow with vortex sheets is beyond the scope of the present project.

There is clearly a need for a truly mathematical formulation which provides all of the characteristic features of a flat-bottomed merchant ship. Section lines must be tangent to the flat bottom. There must be provision for a nearly parallel midbody. There must be the right number of inflection points and regions of maximum curvature in both waterlines and section lines. It is the objective of this report to offer a mathematical formulation which is suitable for global flow over a ship's hull. Not included in the report are formulations for local flows over appendages such as domes, propellers, or rudders.

There are two ways to functionalize a ship hull. In one way the section lines are expressed by simple formulae with parameters which vary lengthwise. In the other way the waterlines are expressed by simple formulae with parameters which vary with elevation. The second way has been found to be more satisfactory, provided seven equally spaced sections were expressed with simple functions. This was enough to establish waterlines at all elevations and section lines at all stations.

Two types of parameters can be recognized. One type is only weakly determinate, and can be expressed therefore by simple formulations. The other type is strongly determinate, and must be expressed as the solution of simultaneous equations.

In the present report there are documentations of programs and subroutines which are specific to ship lines. In other reports there will be documentations[19,20] of subroutines which are of general use for polynomials and matrices.

CARTESIAN COORDINATES

In the simulation of offsets at the surface of the ship it is convenient to use a special coordinate system in which x is the distance backward from the fore perpendicular, y is the offset from the median plane, and z is the distance upward from the flat bottom. Associated with the Cartesian coordinates are unit vectors **i**, **j**, **k** in the directions of increasing coordinates x, y, z.

WATERLINES

In accordance with a fundamental philosophy of ship design, waterlines and section lines are smooth curves with asymptotes which are parallel to the median plane of the ship. Smoothness may be achieved with simple mathematical formulations. The simplest curve with straight asymptotes is the hyperbola, in which case the ordinates are expressed in terms of the abscissae by radicals. The ordinate y for a simple hyperbola is expressed in terms of the abscissa x by the equation

$$y = \alpha \sqrt{\beta^2 + \sigma^2} \tag{1}$$

where the parameter σ is defined by the equation

$$\sigma = x - x_0 \tag{2}$$

while x_0 is the locus of the center of the hyperbola, $\alpha\beta$ is the semidiameter, and α is the slope of the asymptotes.

It is possible to string together a set of hyperbolas with different positions and dimensions to form excellent simulations of conventional waterlines. Let a waterline be represented by the equation

$$y = \alpha_0 + \alpha_1 \sqrt{\beta_1^2 + \sigma_1^2} + \alpha_2 \sqrt{\beta_2^2 + \sigma_2^2} + \alpha_3 \sqrt{\beta_3^2 + \sigma_3^2} + \alpha_4 \sqrt{\beta_4^2 + \sigma_4^2} \tag{3}$$

where the parameters are defined by the equations

$$\sigma_1 = x - x_1 \tag{4}$$

$$\sigma_2 = x - x_2 \tag{5}$$

$$\sigma_3 = x - x_3 \tag{6}$$

$$\sigma_4 = x - x_4 \tag{7}$$

The constant α_0 controls the amount by which the waterline is displaced off the median

plane, and the coefficients $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ control the amplitudes by which the waterline swings in and out. The asymptotes of the curves which are formed by pairs of hyperbolas are parallel to the median plane when the coefficients are related in accordance with the equations

$$\alpha_2 = -\alpha_1 \qquad\qquad \alpha_4 = -\alpha_3 \tag{8}$$

The parameters $\alpha_0, \alpha_1, \alpha_3$ can be computed by the solution of three equations which constrain the offsets to have specified values at the fore perpendicular, at the midship section, and at the aft perpendicular.

The constants $\beta_1, \beta_2, \beta_3, \beta_4$ control the curvature of the waterline in the four regions where the curvature is a maximum. The curvature has fore and aft symmetry when the constants are related in accordance with the equations

$$\beta_1 = \beta_4 \qquad\qquad \beta_2 = \beta_3 \tag{9}$$

The constants have so little effect on the waterline that they are almost arbitrary. They are adjusted by trial to give good simulations of conventional waterlines.

The constants x_1, x_2, x_3, x_4 determine the position and the slope of the waterline near two points of inflection. The constants have a strong effect on the waterline. They can be adjusted by trial until the offsets are equal to specified values at four specified positions along the length of the ship.

An attempt was made to use least squares for the adjustment of constants to a seven-station data set. There was convergence when the data were equally spaced, but there was divergence when the data were moved too close to the fore perpendicular and the aft perpendicular. The rate of convergence or divergence was increased when sequential iteration of the four constants was replaced by the simultaneous evaluation of the constants through inversion of a 4×4 matrix. Thus there are limitations on the range of application of the radical representation.

In a series of experiments the waterlines were simulated with the product of sums of radicals, but this complication was not superior to the simple sum of four radicals.

SECTION LINES

The bend in a section line is expressed by a linear combination of a straight line and a hyperbola. It was found possible to orient line and hyperbola so that one leg of the combination was parallel to the median plane. The bulge in a section line is achieved through the division by another radical. If the constants in both radicals are the same, then the curve is expressed by the sum of a constant and the derivative of a radical as illustrated by the equation

$$\frac{\beta\sigma + \sqrt{1+\beta^2\sigma^2}}{\sqrt{1+\beta^2\sigma^2}} = 1 + \frac{1}{\beta}\frac{d}{dz}\sqrt{1+\beta^2\sigma^2} \tag{10}$$

where the parameter σ is defined by the equation

$$\sigma = z - \alpha \tag{11}$$

and α, β are constants. The curve which is expressed by these functions is S-shaped with horizontal asymptotes. Termination of the curve at the bottom where $z = 0$ is achieved if the functions are multiplied by the factor

$$\left(\frac{z^2}{\epsilon^2 + z^2}\right)^{\frac{1}{4}} \tag{12}$$

where ϵ is a constant. This factor vanishes like the square root of z as $z \to 0$, and approaches unity as $z \to \infty$. The curve which is expressed by the factor is U-shaped with parallel asymptotes. The bottom of the curve osculates a circle if the radius of curvature is equal to unity. Substitution of the factor into Equation (6) in Appendix D shows that the radius of curvature is unity if $\epsilon = \frac{1}{2}$. An opening at the end of waterlines or section lines can be closed with the simulation of a circle if ϵ is scaled to match the width of the opening.

It is possible to combine functions to form excellent simulations of conventional section lines. Let a section line be represented by the equation

$$y = A + \left(B + C\frac{\beta\sigma + \sqrt{1 + \gamma^2\sigma^2}}{\sqrt{1 + \delta^2\sigma^2}}\right)\left(\frac{z^2}{\epsilon^2 + z^2}\right)^{\frac{1}{4}} \tag{13}$$

where the parameter σ is defined by the equation

$$\sigma = z - \alpha \tag{14}$$

The constant α determines the position of the bend. The constants β, γ determine the angle between asymptotes of the bend. The constant δ determines the curvature of the bulge. The constant ϵ determines the curvature of the bilge. The constants A, B, C can be computed by the solution of three equations which constrain the offsets to have specified values at three elevations.

DERIVATIVES

The computation of areas and normals requires the evaluation of partial derivatives. The offset y at the surface is expressed as a function of coordinates x, z on the median plane by the equation

$$y = f(x, z) \tag{15}$$

The function is given by Equation (3) where the parameters $\sigma_1, \sigma_2, \sigma_3, \sigma_4$ are the only functions of x, and the other parameters are functions of z. Differentiation with respect to x is expressed by the equation

$$\frac{\partial f}{\partial x} = \frac{\alpha_1\sigma_1}{\sqrt{\beta_1^2 + \sigma_1^2}} + \frac{\alpha_2\sigma_2}{\sqrt{\beta_2^2 + \sigma_2^2}} + \frac{\alpha_3\sigma_3}{\sqrt{\beta_3^2 + \sigma_3^2}} + \frac{\alpha_4\sigma_4}{\sqrt{\beta_4^2 + \sigma_4^2}} \tag{16}$$

while differentiation with respect to z is expressed by the equation

$$\begin{aligned}\frac{\partial f}{\partial z} = {} & \frac{d\alpha_0}{dz} \\ & + \sqrt{\beta_1^2 + \sigma_1^2}\,\frac{d\alpha_1}{dz} + \sqrt{\beta_2^2 + \sigma_2^2}\,\frac{d\alpha_2}{dz} + \sqrt{\beta_3^2 + \sigma_3^2}\,\frac{d\alpha_3}{dz} + \sqrt{\beta_4^2 + \sigma_4^2}\,\frac{d\alpha_4}{dz} \\ & + \frac{\alpha_1\beta_1}{\sqrt{\beta_1^2 + \sigma_1^2}}\frac{d\beta_1}{dz} + \frac{\alpha_2\beta_2}{\sqrt{\beta_2^2 + \sigma_2^2}}\frac{d\beta_2}{dz} + \frac{\alpha_3\beta_3}{\sqrt{\beta_3^2 + \sigma_3^2}}\frac{d\beta_3}{dz} + \frac{\alpha_4\beta_4}{\sqrt{\beta_4^2 + \sigma_4^2}}\frac{d\beta_4}{dz} \\ & - \frac{\alpha_1\sigma_1}{\sqrt{\beta_1^2 + \sigma_1^2}}\frac{dx_1}{dz} - \frac{\alpha_2\sigma_2}{\sqrt{\beta_2^2 + \sigma_2^2}}\frac{dx_2}{dz} - \frac{\alpha_3\sigma_3}{\sqrt{\beta_3^2 + \sigma_3^2}}\frac{dx_3}{dz} - \frac{\alpha_4\sigma_4}{\sqrt{\beta_4^2 + \sigma_4^2}}\frac{dx_4}{dz}\end{aligned} \tag{17}$$

The derivatives of the parameters can be determined with the solution of a system of seven equations which express the derivatives of the offset at seven stations.

Differentiation in Equation (13) leads to the equation

$$\frac{dy}{dz} = \frac{C}{(1 + \delta^2\sigma^2)^{\frac{3}{2}}}\left[\beta + \frac{(\gamma^2 - \delta^2)\sigma}{\sqrt{1 + \gamma^2\sigma^2}}\right]\left(\frac{z^2}{\epsilon^2 + z^2}\right)^{\frac{1}{4}}$$

$$+ \frac{1}{2}\left[B + C\,\frac{\beta\sigma + \sqrt{1 + \gamma^2\sigma^2}}{\sqrt{1 + \delta^2\sigma^2}}\right]\frac{\epsilon^2}{z^{\frac{1}{2}}(\epsilon^2 + z^2)^{\frac{5}{4}}} \tag{18}$$

which can be used in the determination of the derivatives of the offsets at the seven stations.

The vector area on the surface per unit area on the median plane is given by a vector cross product with the components

$$\left(-\frac{\partial f}{\partial x}, +1, -\frac{\partial f}{\partial z}\right) \tag{19}$$

Although these equations are complicated they must be used for the computation of area unless the radical representation can be replaced accurately by an orthonormal polynomial representation.

ORTHONORMAL POLYNOMIALS

Let $u_1, \cdots, u_n$ be a discrete set of values of a parameter u, and let $\varphi_0(u), \cdots, \varphi_m(u)$ be a set of polynomials of progressively increasing degree. The polynomials are orthonormal if they satisfy the equations

$$\sum_{i=1}^{n} \varphi_j(u_i)\varphi_k(u_i) = \delta_{jk} \tag{20}$$

where δ_{jk} is zero if $j \neq k$ and is unity if $j = k$. The polynomials are generated by the three-term recurrence equation

$$\varphi_k(u) = \frac{1}{\alpha_{k-1}}\left\{u\varphi_{k-1}(u) - \beta_{k-1}\varphi_{k-1}(u) - \alpha_{k-2}\varphi_{k-2}(u)\right\} \tag{21}$$

which is started with the polynomial

$$\varphi_0(u) = \frac{1}{\sqrt{n}} \tag{22}$$

and uses the recurrence constants α_{k-1} and β_{k-1}. The derivatives of the polynomials are generated by the three-term recurrence equation

$$\varphi_k'(u) = \frac{1}{\alpha_{k-1}}\left\{\varphi_{k-1}(u) + u\varphi_{k-1}'(u) - \beta_{k-1}\varphi_{k-1}'(u) - \alpha_{k-2}\varphi_{k-2}'(u)\right\} \tag{23}$$

which is started with the derivative

$$\varphi_0'(u) = 0 \tag{24}$$

The recurrence equations can be used to expand the orthonormal polynomials in

ascending powers of their arguments, but this expansion is inadvisable because there are large terms of intermediate degree which cause loss of accuracy by round off.

Let $c_0, \cdots, c_m$ be the coefficients in a series expansion in terms of the polynomials. An arbitrary function $f(u)$ deviates from the series expansion with a standard deviation σ which is expressed by the equation

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} \left\{ f(u_i) - \sum_{k=0}^{m} c_k \varphi_k(u_i) \right\}^2 \tag{25}$$

Differentiation with respect to the coefficients shows that for least squares deviation the function is represented by the equation

$$f(u) = \sum_{i=1}^{n} \sum_{k=0}^{m} f(u_i) \varphi_k(u_i) \varphi_k(u) \tag{26}$$

and its derivative is given by the equation

$$\frac{d}{du} f(u) = \sum_{i=1}^{n} \sum_{k=0}^{m} f(u_i) \varphi_k(u_i) \varphi_k'(u) \tag{27}$$

The series expansion converges to $f(u)$ with only weak requirements on the continuity of $f(u)$.

ISOMETRIC PARAMETERS

The mapping of a quadrilateral on the median plane is singular at the bottom where the surface is perpendicular to the median plane. There is no such singularity in an isometric mapping. The Cartesian coordinates x, y, z on the surface are expressed as functions of surface coordinates u, v in the map. The surface coordinates are normalized so that they satisfy the inequalities

$$-1 \leq u \leq +1 \qquad\qquad -1 \leq v \leq +1 \tag{28}$$

In the limiting case of a slender ship the coordinate x may be expressed as a linear function of the coordinate u, and the coordinates y, z may be expressed with *orthonormal polynomials* of the coordinate v. Discrete values of v for the expression of coordinates are estimated initially from the perimeter of a polygon which is inscribed in the section line. The metric l along the section line is defined by the equation

$$\frac{dl}{dv} = \sqrt{\left(\frac{dy}{dv}\right)^2 + \left(\frac{dz}{dv}\right)^2} \tag{29}$$

Values of the metric are computed at the discrete values of v, the metric is expressed as a power polynomial in v, and is integrated with respect to v. The discrete values are refined by iteration until the metric is uniform along the section line.

A position vector $\mathbf{r}$ on the surface of the quadrilateral is given by the equation

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \tag{30}$$

A vector surface element on the quadrilateral is given by the cross product between

differential displacements of $\mathbf{r}$ for differential changes in u, v. The vector area $\mathbf{s}$ on the quadrilateral per unit area in the map is given by the equation

$$\mathbf{s} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{dx}{du} & \frac{dy}{du} & \frac{dz}{du} \\ \frac{dx}{dv} & \frac{dy}{dv} & \frac{dz}{dv} \end{vmatrix} \tag{31}$$

The vector $\mathbf{s}$ may be expressed in terms of its components l, m, n by the equation

$$\mathbf{s} = l\mathbf{i} + m\mathbf{j} + n\mathbf{k} \tag{32}$$

The components l, m, n are the cofactors in the determinant of the Equation (31).

Insofar as an arbitrary function can be expressed implicitly in powers of the surface coordinates, the function can be integrated over the map. Seperate integration with respect to each surface coordinate is made possible when the map is subdivided into a grid of lines parallel to the sides of the map and with a spacing proportional to the roots of a Chebyshev polynomial. Then the matrix of integration multipliers for the square map is the matrix product of the arrays of multipliers for line integration. The results of the smooth simulation are three matrices X, Y, Z which list surface coordinates and three matrices L, M, N which list surface components for the grid points in the map.

SIMULATION

The waterlines of the Series 60, Block 0.60 ship configuration have been simulated with the radical representation of Equation (3). The parameters in the representation were adjusted by trial until deviations in offset were optimum in accordance with the Chebyshev criterion. The radicals for a representative waterline are illustrated for comparison in Figure 1.

Offsets were computed at seven stations along the length of the ship from the optimum radical representation. The offsets at the seven stations were simulated with the section line representation of Equation (13). The parameters in the representation were adjusted by trial and by least squares. Further adjustments were necessary in the aft body in order to obtain fair section lines. Efficient trial computations were made possible by an HP67 programmable pocket computer. The final parameters in the smooth simulation are tabulated in Appendix A.

The deviations in offset between the tabular data and the final simulation are illustrated in Figure 2. There is good agreement near the bow, but there is disagreement near the stern, because the stern post has been moved aft to the aft perpendicular. Any residual discrepancy can be considered as a local disturbance in the global configuration.

The final formulation was used in the construction of a closely spaced table of offsets, which is illustrated by points in Figures 3 and 4. The table of offsets was converted into an orthonormal polynomial representation, which is illustrated by curves in Figures 3 and 4. There is accurate agreement between points and curves.

PROGRAMMING

Transformations of ship lines between tabular formats and functional formats is accomplished through the concatenation of subroutines and programs.

The parameters in the radical representation are normalized to give specific offsets at three stations by the following subroutine.

```
SUBROUTINE NWLN (XN, AX, AA, AB, YN)
```

****

FORTRAN SUBROUTINE FOR NORMALIZATION OF WATERLINE

****

The longitudinal coordinates of three stations are given in the 3-array XN. The parameters x_1, x_2, x_3, x_4 are given in the 4-array AX. The parameters $\beta_1, \beta_2, \beta_3, \beta_4$ are given in the 4-array AB. The offsets at the three stations are given in the 3-array YN. New coefficients are computed by the solution of three linear equations. The coefficients $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ are stored in the 5-array AA.

The offset on a waterline is computed by reference to the following subroutine.

```
SUBROUTINE WLNO (X, AX, AA, AB, Y)
```

****

FORTRAN SUBROUTINE FOR WATERLINE OFFSET

****

The longitudinal coordinate x is given in the argument X. The parameters x_1, x_2, x_3, x_4 are given in the 4-array AX. The coefficients $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ are given in the 5-array AA. The parameters $\beta_1, \beta_2, \beta_3, \beta_4$ are given in the 4-array AB. The offset is computed from the radical representation. The offset y is stored in the function Y.

The derivative of the offset on a waterline is computed by reference to the following subroutine.

```
SUBROUTINE WLND (X, AX, AA, AB, D)
```

****

FORTRAN SUBROUTINE FOR DERIVATIVE OF WATERLINE OFFSET

****

The longitudinal coordinate x is given in the argument X. The parameters x_1, x_2, x_3, x_4 are given in the 4-array AX. The coefficients $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ are given in the 5-array AA. The parameters $\beta_1, \beta_2, \beta_3, \beta_4$ are given in the 4-array AB. The derivative is computed from the radical representation. The derivative dy/dx is stored in the function D.

The parameters in the section line representation are adjusted to give a specific offset at one waterline by the following subroutine.

```
SUBROUTINE NSLN (ZN, CC, YN)
```

****

FORTRAN SUBROUTINE FOR NORMALIZATION OF SECTION LINE

****

The elevation coordinate of the waterline is given in the argument ZN. The parameters $\alpha, \beta, \gamma, \delta, \epsilon, A, B, C$ are given in the 8-array CC. The offset at the waterline is given in the argument YN. The parameter C is adjusted to make the computed offset equal to the given offset.

The offset on a section line is computed by reference to the following subroutine.

```
SUBROUTINE SLNO (Z, CC, Y)
```

****

FORTRAN SUBROUTINE FOR SECTION LINE OFFSET

****

The elevation coordinate z is given in the argument Z. The parameters $\alpha, \beta, \gamma, \delta, \epsilon, A, B, C$ are given in the 8-array CC. The offset y is stored in function Y.

The derivative of the offset on a section line is computed by reference to the following subroutine.

```
SUBROUTINE SLND (Z, CC, D)
```

****

FORTRAN SUBROUTINE FOR DERIVATIVE OF SECTION LINE OFFSET

****

The elevation coordinate z is given in the argument Z. The parameters $\alpha, \beta, \gamma, \delta, \epsilon, A, B, C$ are given in the 8-array CC. The derivative dy/dz is stored in function D.

The transformation of offset data from functional format to tabular format is accomplished by reference to the following program.

```
PROGRAM SMSM (INPUT, TAPE7, OUTPUT)
```

****

FORTRAN PROGRAM FOR SMOOTH SIMULATION

****

The parameters for section lines at seven stations are given by DATA statements. The x-coordinates and the z-coordinates for a table of offsets are given by DATA statements. A matrix of partial derivatives of offset with respect to parameters for each waterline is derived from small increments in the parameters. Inversion of a succession of matrices brings into agreement the waterline offsets and the section line offsets at the seven stations. The table of offsets is written on TAPE7.

The waterlines of the ship are compared with the data from which they are derived by reference to the following program.

```
PROGRAM LINES (INPUT, OUTPUT, TAPE1, TAPE7)
```

****

FORTRAN PROGRAM TO PLOT OFFSET DATA AND WATERLINES

****

The x-coordinates of section lines and the z-coordinates of waterlines are given in DATA statements. The offset data are given on TAPE7. Noise in the offset data is reduced to least squares by orthonormal polynomial representation. The waterlines are forced to pass through the end points of the data by a linear adjustment. Plot instructions for the CalComp plotters are recorded on TAPE1.

The section lines of the ship are compared with the data from which they are derived by reference to the following program.

```
PROGRAM BODY (INPUT, OUTPUT, TAPE1, TAPE7)
```

****

FORTRAN PROGRAM TO PLOT OFFSET DATA AND SECTION LINES

****

The x-coordinates of section lines and the z-coordinates of waterlines are given in DATA statements. The offset data are given on TAPE7. Noise in the offset data is reduced

to least squares by orthonormal polynomial representation. The section lines are forced to pass through the end points of the data by a linear adjustment. Plot instructions for the CalComp plotters are recorded on TAPE1.

DISCUSSION

The least squares convergence of a polynomial to a radical is rapid only within the range of convergence of a Taylor series expansion. The slow convergence of the polynomial representation limits its range of application. The difference between representations is exemplified by their behavior with increasing arguments. The polynomials increase indefinitely with an increasing rate, whereas the radicals approach asymptotes of finite slope.

Once one smooth simulation has been achieved it is possible to generate an infinity of other smooth simulations with the scaling of breadth and draft. Changes in block coefficient can be achieved with shifts in the centers of the hyperbolas.

The use of radicals has been successful for the fairing of a merchant ship with a flat bottom. The use of radicals looks promising for the fairing of a complete range of ships from tankers with blunt bows to warships with sharp bows. Representative ship plans are in hand, and some trial computations will be undertaken if time permits. There will not be time in the present project for a thorough investigation.

CONCLUSION

A radical representation has been successful for the smooth simulation of a merchant ship. The possibility of using radicals in the fairing of tankers and warships should be investigated further.

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APPENDIX A

PARAMETERS

TABLE I

WATERLINES AT SEVEN ELEVATIONS

WL	x_1	x_2	x_3	x_4
0.00	3.900000	8.800000	11.300000	17.800000
0.25	0.916690	7.708895	12.794486	19.024116
0.50	0.910125	6.853267	14.222032	18.746849
0.75	0.982791	6.375124	15.383269	19.052597
1.00	0.854743	6.049528	16.336156	19.942718
1.25	0.297610	5.809324	17.001219	21.032526
1.50	−2.000000	5.600000	18.000000	22.200000

WL	α_0	α_1	α_2	α_3	α_4
0.00	−0.004390	0.074161	−0.074161	−0.056375	0.056375
0.25	−0.142102	0.084603	−0.084603	−0.092790	0.092790
0.50	−0.128860	0.097260	−0.097260	−0.128410	0.128410
0.75	−0.148512	0.106981	−0.106981	−0.163441	0.163441
1.00	−0.173393	0.111567	−0.111567	−0.171716	0.171716
1.25	−0.144701	0.107520	−0.107520	−0.142895	0.142895
1.50	−0.302918	0.093630	−0.093630	−0.146412	0.146412

WL	β_1	β_2	β_3	β_4
0.00	1.330000	1.330000	1.330000	1.330000
0.25	2.657353	1.856834	1.856834	2.657353
0.50	2.370394	1.949367	1.949367	2.370394
0.75	2.323569	1.978622	1.978622	2.323569
1.00	2.309238	1.990668	1.990668	2.309238
1.25	2.303136	1.996640	1.996640	2.303136
1.50	2.300000	2.000000	2.000000	2.300000

TABLE II

SECTION LINES AT SEVEN STATIONS

Station	α	β	γ	δ
FP	1.200000	4.000000	4.000000	0.000000
$3\frac{1}{3}$	1.199377	1.892324	1.892324	0.000000
$6\frac{2}{3}$	1.197513	0.480115	0.480115	0.000000
10	1.188297	0.000000	0.000000	0.000000
$13\frac{1}{3}$	1.042503	0.191434	0.191434	1.182753
$16\frac{2}{3}$	0.645218	3.023980	3.023980	1.547807
AP	0.950000	10.000000	10.000000	1.280000

Station	ϵ	A	B	C
FP	0.100000	0.006000	0.000000	0.013000
$3\frac{1}{3}$	0.276175	0.049243	0.326458	0.117558
$6\frac{2}{3}$	0.317775	0.397180	0.446322	0.087418
10	0.167000	0.667000	0.333000	0.000000
$13\frac{1}{3}$	0.502309	0.477512	0.533646	0.001716
$16\frac{2}{3}$	0.259881	0.142657	0.213017	0.179816
AP	0.100000	0.006000	0.000000	0.050000

APPENDIX B

OFFSETS

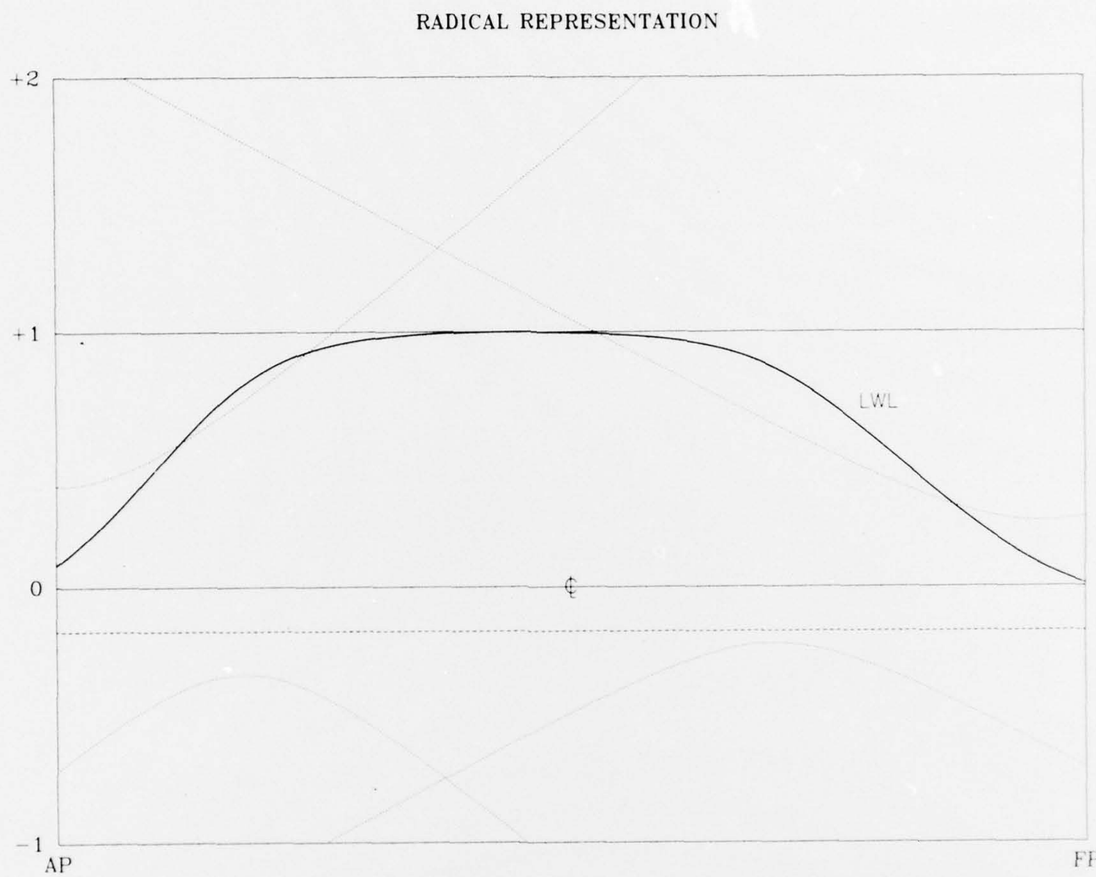

Figure 1. Hyperbolas for the Radical Representation of a Waterline. ---, constant offset; ···, radical offsets; —, waterline offset.

WATERLINE OFFSETS

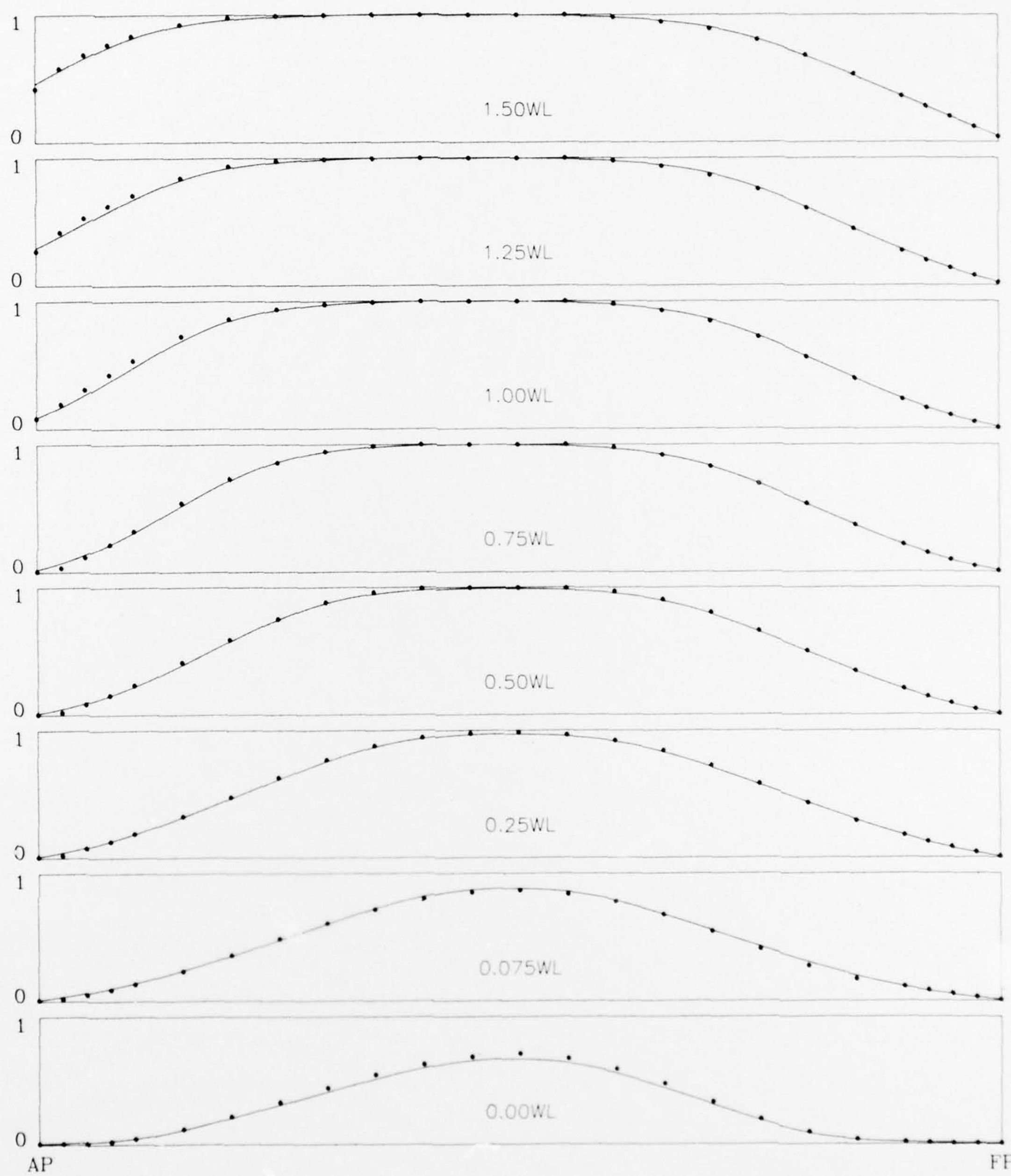

Figure 2. Comparisons between Tabular Data and Smooth Simulations. •, tabular offsets from NSRDC Report No. 1712; –, waterline offsets of the smooth simulations.

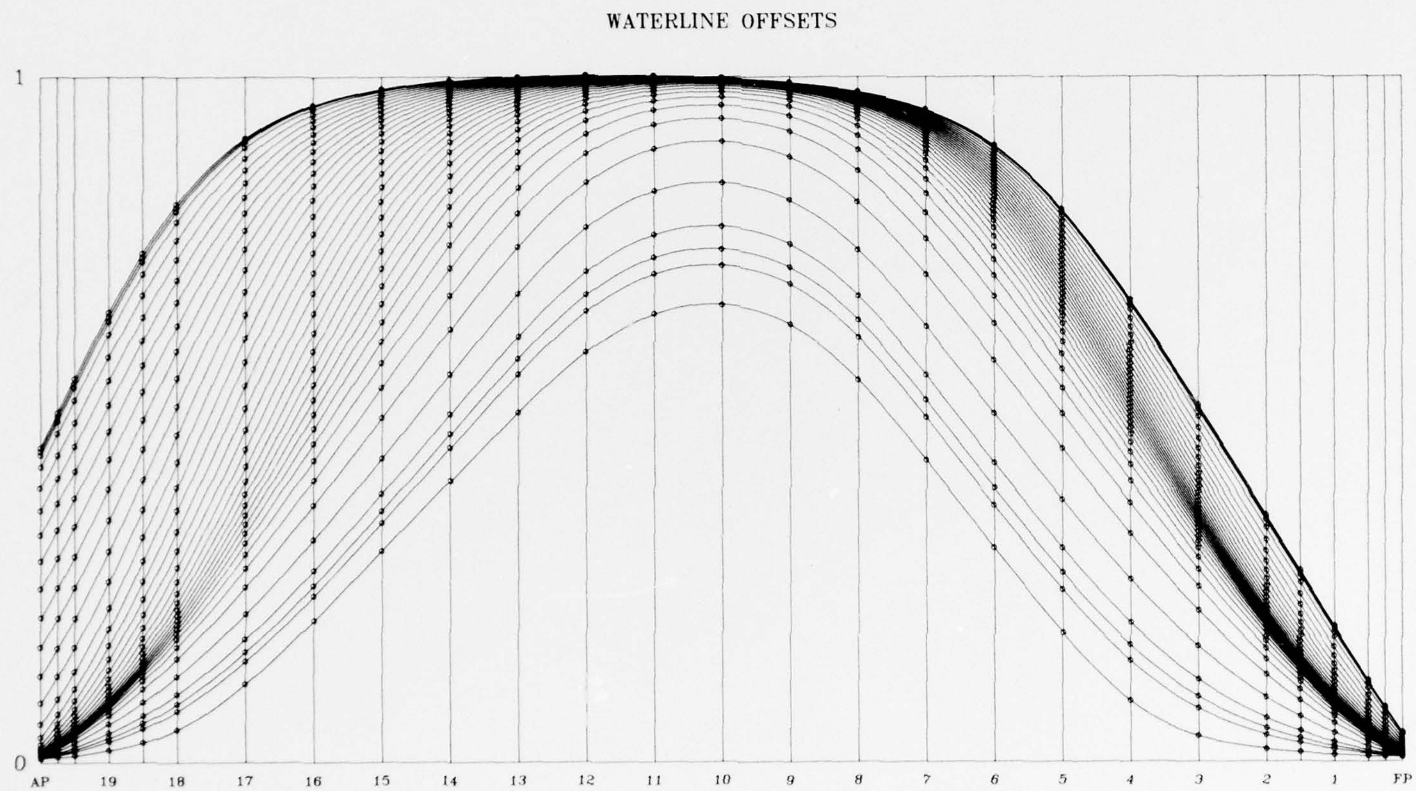

Figure 3. Waterlines of Smooth Simulation. o, radical representation; –, orthonormal polynomial representation. Elevations of the waterlines are indicated in Figure 4.

BODY PLAN

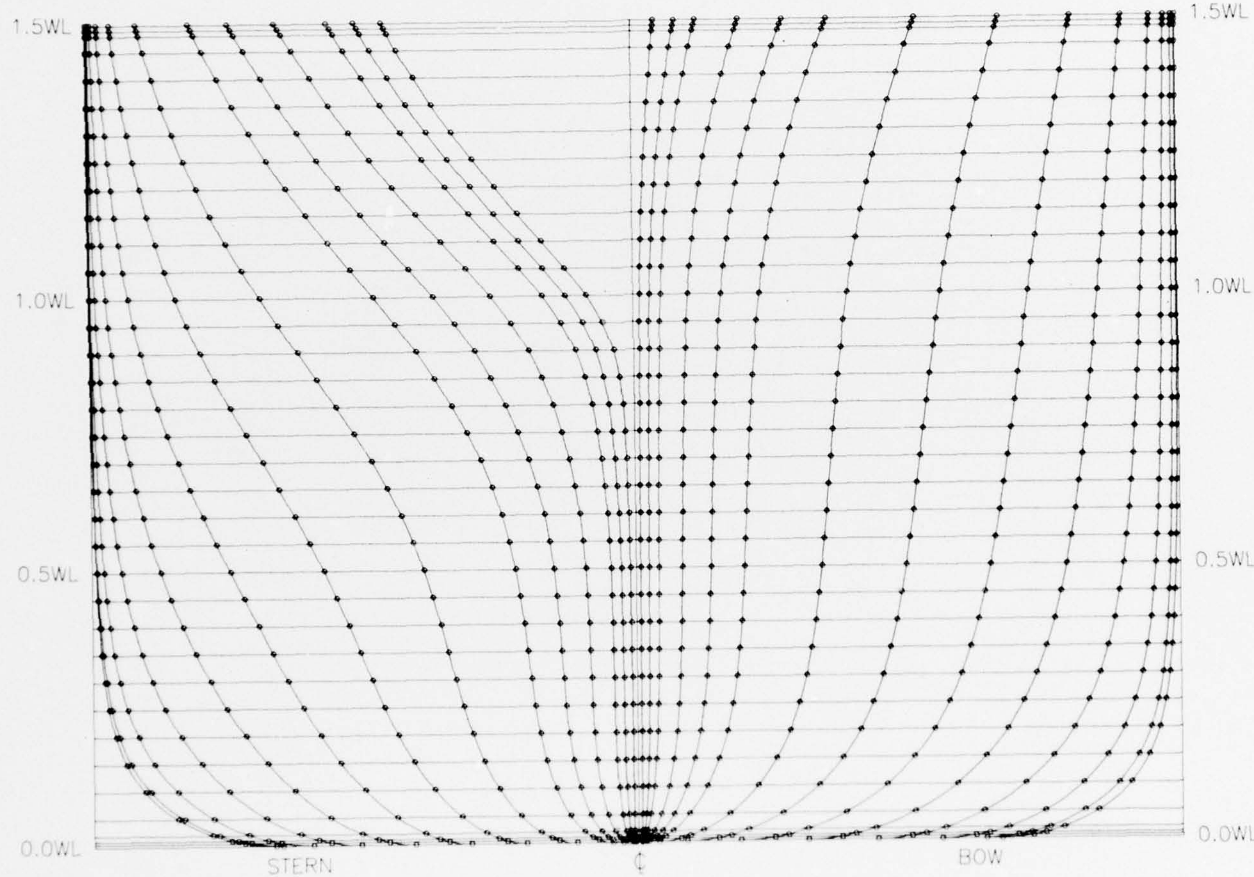

Figure 4. Section Lines of Smooth Simulation. ○, radical representation; –, orthonormal polynomial representation. Stations of the section lines are indicated in Figure 3.

APPENDIX C

DIHEDRAL FLOW

DIHEDRAL FLOW

That the velocity is infinite for potential flow over a convex dihedral angle can be understood from an analysis in terms of complex variables. Let w be a complex potential which is expressed in terms of its real and imaginary parts by the equation

$$w = u + iv \tag{1}$$

Let z be a complex position which is expressed in terms of its real and imaginary parts by the equation

$$z = x + iy \tag{2}$$

If w is an analytic function of z, then dw/dz is independent of the direction of dz, and u, v satisfy the Cauchy–Riemann equations,

$$\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y} \tag{3}$$

Differentiation shows that u, v satisfy the Laplace equations,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0 \tag{4}$$

If dz is a line element in the z-plane, then the flux of $-\nabla u$ across a line with normal $i\,dz$ is the change in v along the line element dz. Thus if u is a velocity potential such that velocity is the negative gradient of u, then v is the stream function such that the flux between streamlines is the difference in v.

Let w be expressed by the equation

$$w = i\,z^{\gamma} \tag{5}$$

where γ is a noninteger exponent. By Euler's theorem the complex parts are connected by the equation

$$u + iv = -r^{\gamma}\sin\gamma\phi + i\,r^{\gamma}\cos\gamma\phi \tag{6}$$

where r, ϕ are cylindrical polar coordinates such that

$$x = r\cos\phi \qquad y = r\sin\phi \tag{7}$$

then the stream function is zero where

$$\gamma\phi = \tfrac{1}{2}\pi \tag{8}$$

For a convex dihedral angle, the range of ϕ is more than $\frac{1}{2}\pi$ and γ is less than unity. Since derivatives with respect to r contain the factor $r^{\gamma-1}$, the derivatives become infinite as $r \to 0$.

It is not necessary for the fluid to adhere to the dihedral boundary in an ideal fluid. In a more realistic flow there would be separation with the origination of a vortex sheet from the vertex of the dihedral angle. Imagine what a network of vortex sheets would trail behind a polyhedron!

APPENDIX D

PHYSICAL SPLINES

SPLINE CURVE

The shape of the nonlinear spline is computed with finite difference techniques by Malcolm[10]. The shape of the physical spline is given preferably by elliptic integrals for which there are efficient subroutines.

The spline is a long slender elastic bar. It is bent into a curve by the application of forces and torques to its ends. Let x, y be the Cartesian coordinates of a point on the curve. The shape of the curve is determined by the equilibrium of forces and torques on each differential element of the bar. Let the curve be divided into segments of length dl. Equilibrium requires that the net force on any segment be zero, while equality of action and reaction requires that the forces across a section shall be equal in magnitude but opposite in sign. Let f be the force which one side of the bar exerts on the segment of length dl. Then f also is the force which the segment of length dl exerts on the other side of the bar.

Let the constant force f be directed along the y-axis. The force f exerts a torque of magnitude $f\,dx$ on the segment of length dl. The torque must be balanced by a difference dM in the moments which are applied to the ends of the segment. The moment M is proportional to the curvature of the bar.

If no force is applied to the bar, then the moment M is constant. The bar is bent into an arc of a circle.

If a force is applied to the bar, then the moment M is a linear function of x. Let the origin of x be located at a point where M is zero. Then the moment M is given by the equation

$$M = fx \tag{1}$$

The moment M is given in terms of the curvature of the bar by the equation

$$M = K\frac{d\theta}{dl} \tag{2}$$

where K is the bending modulus of the bar, and θ is the angle which the tangent to the curve makes with the x-axis.

The angle θ is defined by the equation

$$\theta = \tan^{-1}\frac{dy}{dx} \tag{3}$$

Differentiation with respect to x leads to the equations

$$\frac{d\theta}{dx} = \frac{\dfrac{d^2y}{dx^2}}{1 + \left(\dfrac{dy}{dx}\right)^2} \tag{4}$$

and

$$\frac{dl}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \tag{5}$$

while the radius of curvature R is given by the well-known equation

$$\frac{1}{R} = \frac{d\theta}{dl} = \frac{\frac{d^2y}{dx^2}}{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}} \tag{6}$$

The spline curve is determined therefore by the equation

$$\frac{\frac{d^2y}{dx^2}}{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}} = \frac{f}{K}x \tag{7}$$

Integration leads to the equation

$$\frac{\frac{dy}{dx}}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}} = a + cx^2 \tag{8}$$

where a is an arbitrary constant and c is defined by the equation

$$c = \frac{f}{2K} \tag{9}$$

Solution for dy/dx and integration lead to the equation

$$y = \int \frac{a + cx^2}{\sqrt{1 - a - cx^2}\sqrt{1 + a + cx^2}}\,dx \tag{10}$$

The substitutions

$$x = \left(\frac{1-a}{c}\right)^{\frac{1}{2}} \cos\phi \tag{11}$$

and

$$k = \left(\frac{1-a}{2}\right)^{\frac{1}{2}} \tag{12}$$

convert the integration into elliptic integrals as expressed by the equation

$$y = \frac{1}{\sqrt{2c}}F(\phi, k) - \sqrt{\frac{2}{c}}E(\phi, k) \tag{13}$$

where the elliptic integrals of the first and second kinds are defined by the equations

$$F(\phi, k) = \int_0^\phi \frac{d\varphi}{\sqrt{1 - k^2\sin^2\varphi}} \tag{14}$$

$$E(\phi, k) = \int_0^\phi \sqrt{1 - k^2\sin^2\varphi}\,d\varphi \tag{15}$$

The elliptic integrals give the locus of a free elastic spline which is subject to forces alone at its ends.

CALCULUS OF VARIATIONS

Let x be an independent variable and let y, y', y'' be a dependent variable and its derivatives with respect to x. Let $f(x, y, y', y'')$ be the integrand of the integral

$$\int_a^b f(x, y, y', y'')\, dx \tag{16}$$

where a, b are fixed limits of integration. Let the integral be stationary with respect to any arbitrary variation as expressed by the equation

$$\delta \int_a^b f(x, y, y', y'')\, dx = 0 \tag{17}$$

Variation in the integral is derived from variation in y, y', y'' as expressed by the substitutions

$$y \to y + \delta y \tag{18}$$

$$y' \to y' + \frac{d}{dx}\,\delta y \tag{19}$$

$$y'' \to y'' + \frac{d^2}{dx^2}\,\delta y \tag{20}$$

Substitution in the integral leads to the equation

$$\delta \int_a^b f(x, y, y', y'')\, dx = \int_a^b \left\{ \frac{\partial f}{\partial y}\,\delta y + \frac{\partial f}{\partial y'}\,\delta y' + \frac{\partial f}{\partial y''}\,\delta y'' \right\} dx \tag{21}$$

and integration by parts leads to the equation

$$\begin{aligned}\delta \int_a^b f(x, y, y', y'')\, dx = \int_a^b &\left\{ \left(\frac{\partial f}{\partial y}\right) - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) + \frac{d^2}{dx^2}\left(\frac{\partial f}{\partial y''}\right) \right\} \delta y\, dx \\ &+ \left(\frac{\partial f}{\partial y'}\right)\delta y\,\Big|_a^b - \frac{d}{dx}\left(\frac{\partial f}{\partial y''}\right)\delta y\,\Big|_a^b + \left(\frac{\partial f}{\partial y''}\right)\delta y'\,\Big|_a^b \end{aligned} \tag{22}$$

Thus if $\delta y, \delta y'$ are zero at a, b then the integral is stationary for any arbitrary δy only if f is a solution of the Euler differential equation

$$\left(\frac{\partial f}{\partial y}\right) - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) + \frac{d^2}{dx^2}\left(\frac{\partial f}{\partial y''}\right) = 0 \tag{23}$$

Solution of the differential equation determines the relationship between x and y for a stationary integral.

MINIMUM CURVATURE

In the case of the integration of the square of the curvature along the length of a curve the integral is

$$\int_a^b \frac{(y'')^2}{\{1 + (y')^2\}^{\frac{5}{2}}}\, dx \tag{24}$$

Inasmuch as the integrand does not depend upon y, the Euler equation can be integrated to give the equation

$$-\frac{\partial f}{\partial y'} + \frac{d}{dx}\left(\frac{\partial f}{\partial y''}\right) = \text{constant} \tag{25}$$

Substitution leads to the equation

$$\frac{2y'''}{\{1+(y')^2\}^{\frac{5}{2}}} - \frac{5y'(y'')^2}{\{1+(y')^2\}^{\frac{7}{2}}} = \text{constant} \tag{26}$$

If this equation is multiplied throughout by y'', it may be integrated to give the equation

$$\frac{(y'')^2}{\{1+(y')^2\}^3} = \frac{A+By'}{\sqrt{1+(y')^2}} \tag{27}$$

where A, B are arbitrary constants. Let the coordinate axes be oriented so that the x-axis is in the direction of the vector $A\mathbf{i} + B\mathbf{j}$. Let the angle θ be the angle between the tangent of the curve and the x-axis. Then the angle θ satisfies the equation

$$\left(\frac{d\theta}{dl}\right)^2 = C \sin\theta \tag{28}$$

where C is the magnitude of the vector $A\mathbf{i} + B\mathbf{j}$. The x-coordinate is related to the distance l along the curve by the equation

$$\frac{dx}{dl} = \cos\theta \tag{29}$$

Substitution and integration leads to the equation

$$2\sin^{\frac{1}{2}}\theta = C^{\frac{1}{2}}x \tag{30}$$

where the origin of x is selected to be that point where θ is zero. Squaring of both sides of the equation leads to the equation

$$\sin\theta = \frac{y'}{\sqrt{1+(y')^2}} = \tfrac{1}{4}Cx^2 \tag{31}$$

Comparison of this equation with the equations for the physical spline shows that the curve of minimum square of the curvature is a special case of the elastic curve.

APPENDIX E

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Waterlines	Isometric Parameters
Section Lines	Simulation
Derivatives	Programming

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Straight lines, circles, and splines are used in most mathematical representations of ship lines. The representations are only piecewise analytic. Discontinuities in curvature occur along waterlines and section lines. The discontinuities upset computations of velocity distribution in the flow around the ship. The discontinuities are smoothed when the lines are approximated by orthonormal polynomials. The discontinuities are eliminated when the lines are simulated with the aid of radicals. A smooth simulation of the Series 60, Block 0.60 ship model is given by a radical representation.

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